EUREX Architects of trusted markets

Research paper

Finding Delta in the depths of history

Fixed Income ETF Options – Finding Delta in the depths of history

Content

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Management Summary

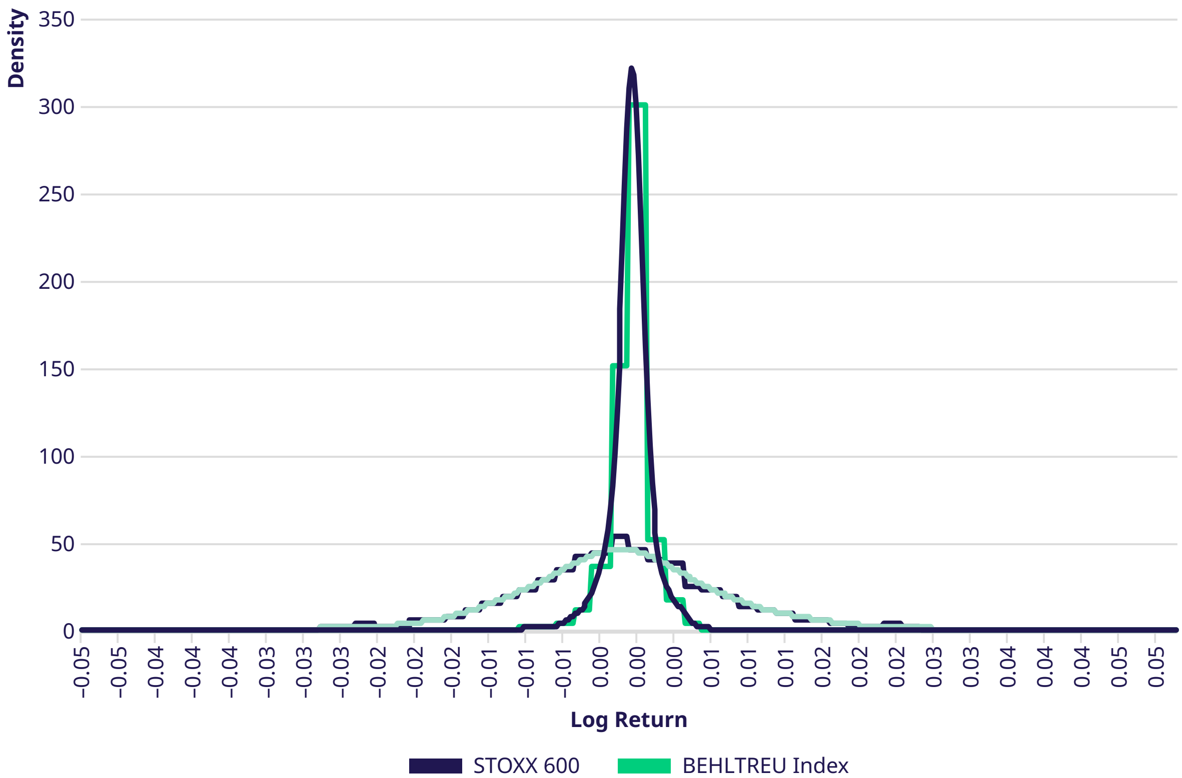
- **Fixed income options with high implied volatility can still be cheap versus the empirical distribution of returns.**
- **Options implied volatility needs to be evaluated versus its realized returns distribution.**
- **Asymmetric returns in fixed income elevate implied volatility skew for out-of-the-money put options.**

Options are widely traded instruments in many asset classes. Across Equities, FX, Fixed Income and Commodities, investors use these to express views on the market and manage their exposures. Options allow the holder to buy (call) or sell (put) the underlying for a predetermined price (strike); however, there is no requirement to exercise, so the payoff of the claim is always non-negative (but can indeed be 0).

Options are priced based on volatility expectations and have a positive value (premium). Here, they differ from linear instruments like futures, which are priced so that they have zero present value. Options remove one side of the distribution of the underlying, and the investor can only gain. Higher volatility means greater upside potential. In fact, many market participants quote options in terms of the implied volatility that would generate the premium they envision in a Black-Scholes model.

The link between implied volatility and premium goes even deeper. Market Makers can hedge their short options by entering an offsetting position in the underlying market, where the ratio (Delta) is set based upon how likely the option is going to be in the money. But the Delta changes over time, depending on volatility and where the underlying is relative to the strike. In fact, the holder of a short option always needs to buy the underlying when it appreciates and sell when it depreciates. In the end, the hedger has to buy high and sell low. With this view, the premium can be understood as the implied costs of managing the hedge. Higher volatility means more adjusting of the hedges, hence higher premium.

Figure 1: Histogram of Log returns for broad market Indices in Equities (STOXX® 600) and EUR HY Corporate Bonds (BEHLTREU)



Source: Bloomberg, Eurex calculations

But when is an option cheap and when is it expensive? In the equity space, investors often look at how the quoted volatility compares with realized price volatility over the past few months. Here, market participants assume that the returns distribution is roughly consistent with the Black-Scholes model, as the implied volatility is derived from the model. In fact, this assumption is relatively in line with reality, as the historical returns density is broadly similar to what you would expect from a normal distribution, at least for medium timeframes. Returns are not skewed towards the positive or negative, and while the far tails are a bit fatter than what one would expect, in general, the kurtosis is well-behaved.

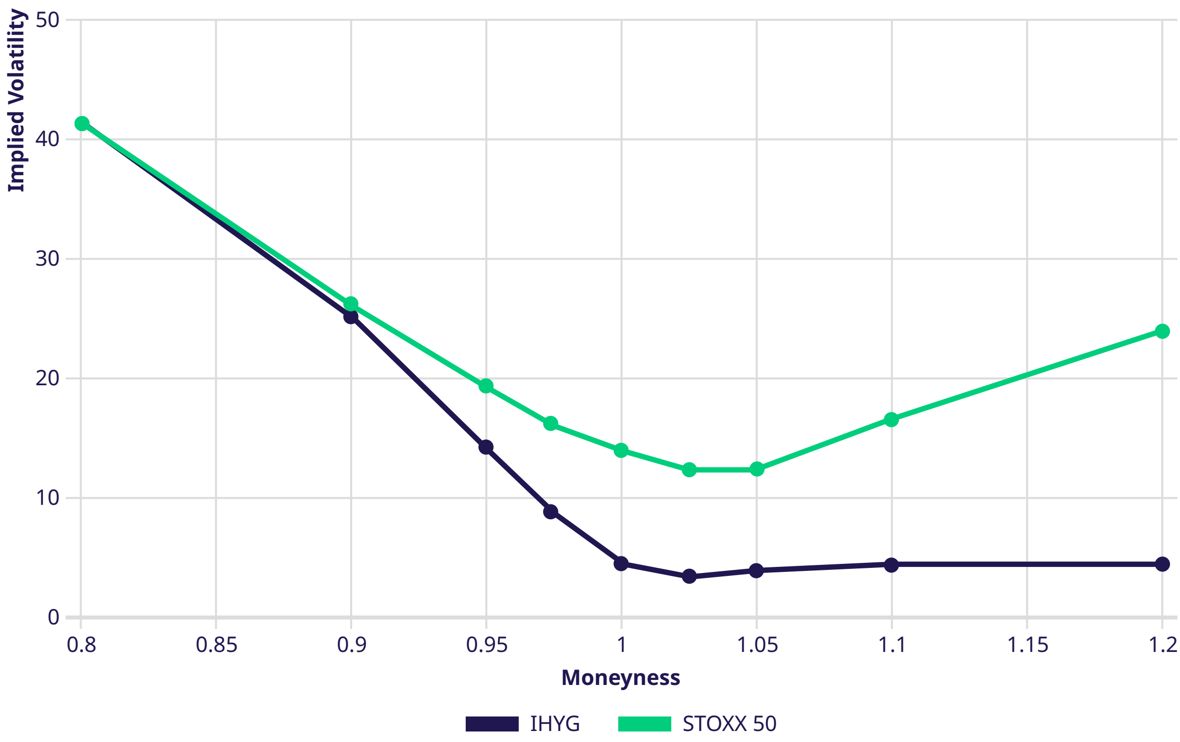
This is different for fixed income underlyings. Returns to the upside are somewhat limited through the fixed amount of cash flows. However, the downside risk still prevails, as issuers can still default. This leads to a distribution with a lot less variance to begin with, but with fatter left tails and skew, with returns that struggle to be squashed into the Black-Scholes framework.

In practice, we observe that the smile of implied volatility is very much skewed to the downside as well. Out-of-the-money puts can command volatility premia of 2–3x the at-the-money volatility. This makes them look significantly more expensive than their equity counterparts, once we normalize for volatility. Intuitively, this makes sense given the distribution we discussed above. If the risk is skewed towards one side, then the option that limits its exposure to this side should also trade richer than calls on the other side.

So, how do you evaluate option prices in an environment where the standard model does not really apply? What does the difference between implied and realized volatility even mean in this context? One way to address these questions is to step back from big model assumptions and focus on what we can directly infer from the market holistically. While we don't know what the real returns distribution looks like, we definitely know how the market is pricing various options by observing quotes in the order book.

Indeed, option prices for various strikes let us directly reconstruct the implied distribution for the returns. Specifically, we can infer the market implied risk-neutral-density of a stretch of the returns distribution by observing the pricing of options of two neighboring strikes. These spreads eliminate all the price movements beyond the narrow gap between the strikes. If we divide the distance in price by the distance in strike, we get the average cumulative density over the interval. The closer the strikes are together, the more exactly this differential should reflect the cumulative density. The density of the distribution is then just the incremental change between the cumulative densities.

Figure 2: 1M Implied Volatility smile for IHYG and SX5E Options on 17.10.2025



For Calls (c), we have (Options on futures here for brevity)

$$c(K) = \int_k^{\infty} (F-K)\,dP_F$$

Hence, the finite difference is

$$\frac{c(K)-c(K+h)}{h} = \frac{h\int_{K+h}^{\infty} dP_x + \int_K^{K+h} (F-K)\,dP_F}{h}$$

$$\leq \int_{K+h}^{\infty} dP_F + \frac{\int_K^{K+h} h\, max_{F\in[K,K+h]}\, dP_F}{h}$$

$$\leq \int_{K+h}^{\infty} dP_F + ch$$

So, in the limit we see

$$\lim_{h\to 0} \frac{C(c)-c(K+h)}{h} = \int_K^{\infty} dP_F = P(F>K)$$

This is exactly the probability that an underlying will be in the money.

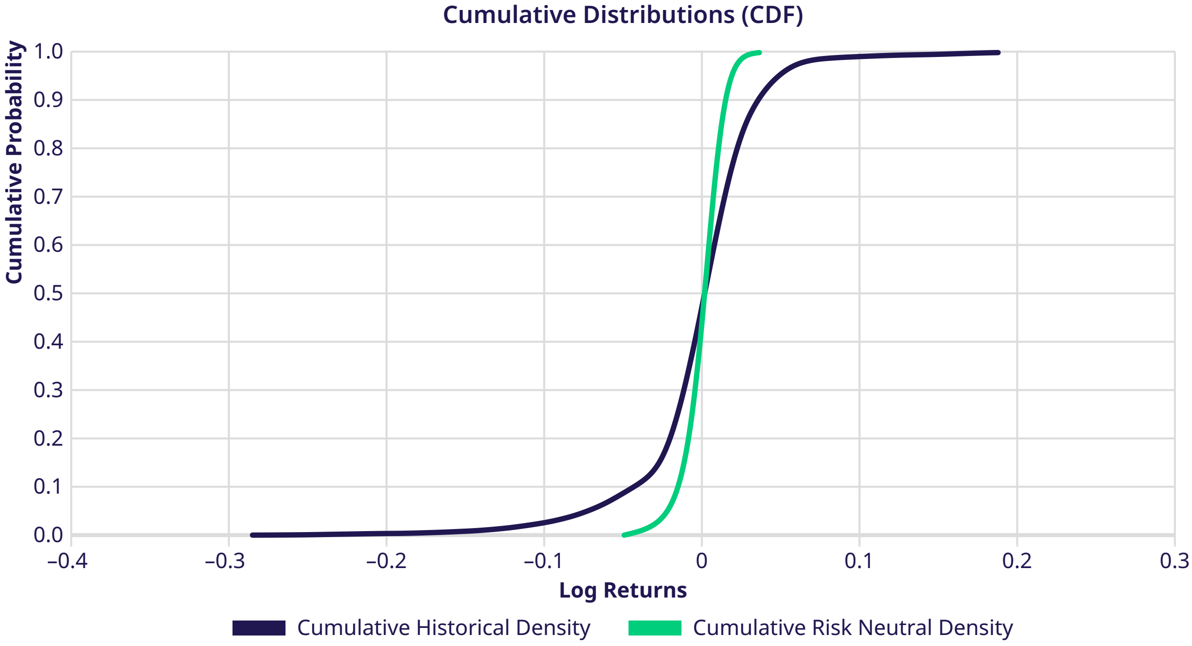
The Risk-Neutral-Distribution in contact with reality

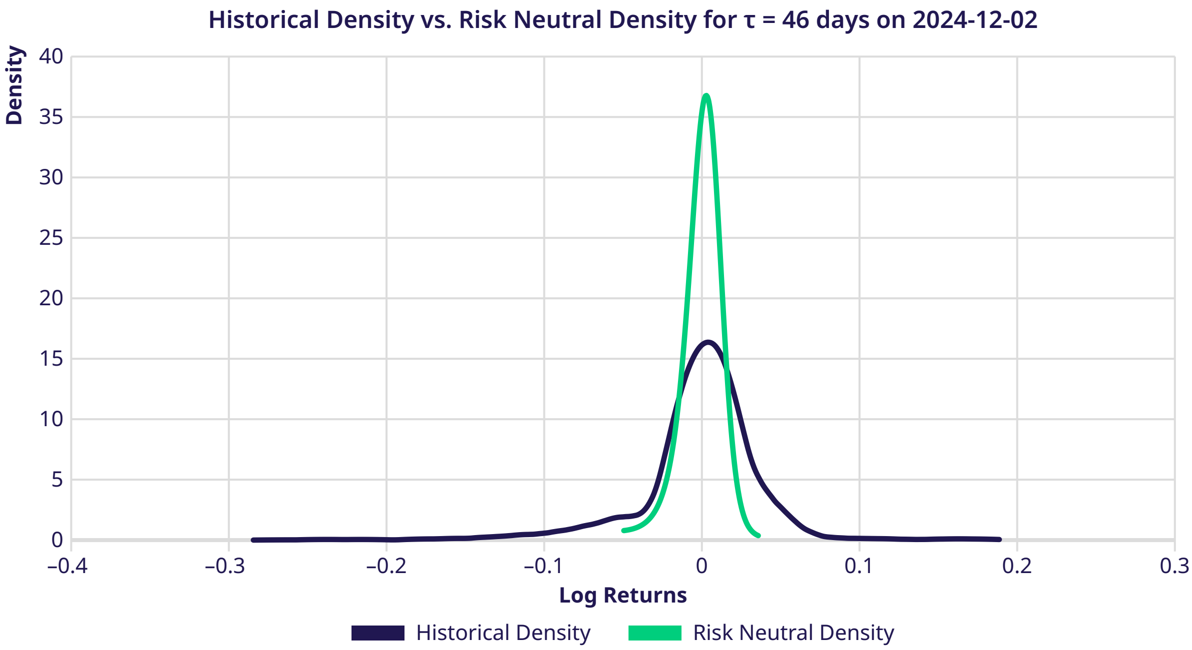
Let's take stock of what we have done so far. Instead of trying to fit fixed income assets into the mold of a model that does not fit their returns, we have instead looked at the market pricing of the whole options market and inferred the distribution from there. This is especially useful for our fixed income underlyings where the returns distribution is so far removed from what the Black-Scholes assumptions would dictate.

So how does that help us to see if the option we want to trade is priced attractively? Well, we see exactly how likely the market implies the price to fall within a range and can compare that to our benchmarks. For example, one could look at the empirical distribution of returns and compare it to the implied distribution.

If the market is pricing the cumulative risk-neutral density of the strike of an out-of-the-money put at 5%, but the empirical cumulative density of the return is actually 10%, then the option is priced cheaply versus its history.

Figure 3: Implied and Historical Kernel-Density-Estimated Distribution of EUR HY ETF (IHYG) Options





Source: Bloomberg, Eurex calculations

The empirical distribution we compare with has to take into account future cash flows generated by the underlying. We have so far compared the risk-neutral density with the empirical distribution of the underlying. But, we have to be careful here, specifically when we think about the empirical distribution. The RND is a distribution of forward values of the underlying, and hence, forward returns, not of the spot market. The securities may still generate total return on the way to expiry. The option is, however, on the underlying, and its final value depends on where this is trading at expiry, not the total return. Following the cost-of-carry approach, the forward price of the underlying is mostly based on how much money you have to borrow to buy it, the associated financing rate and how much cash flows it generates until expiry in terms of securities lending and coupons/dividends.

With this understanding, we see that out-of-the-money puts on fixed income underlyings are actually often underpriced versus history, particularly when considering deep out-of-the-money options.

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